Bulgarian Academy of Sciences. Space Research Institute.
Aerospace Research in Bulgaria. 21, 2007, Sofia

A METHOD OF PHASE-MANIPULATED COMPLEMENTARY SIGNALS APPLIED IN SPACECRAFT-BASED RADARS

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Abstract

Radar imagery, realized by means of synthetic aperture radars (SARs) is very important in exploring planet, satellite and comet surfaces. The most valuable feature of the autocorrelation function (ACF) of the SAR signals is the level of their side lobes, because they determine the dynamic range of the image and the possibility to discover small objects. With regard to this, our paper suggests a method for applying in spacecraft-based SARs the so named generalized complementary signals, whose ACF does not have any side-lobes. It uses the polarization features of electromagnetic waves.

1. Introduction

Radar imagery is very important in exploring planet, satellite and comet surfaces [1, 2]. It may be sketched as follows. The transmitter of the spacecraft-based radar sends electromagnetic signals.

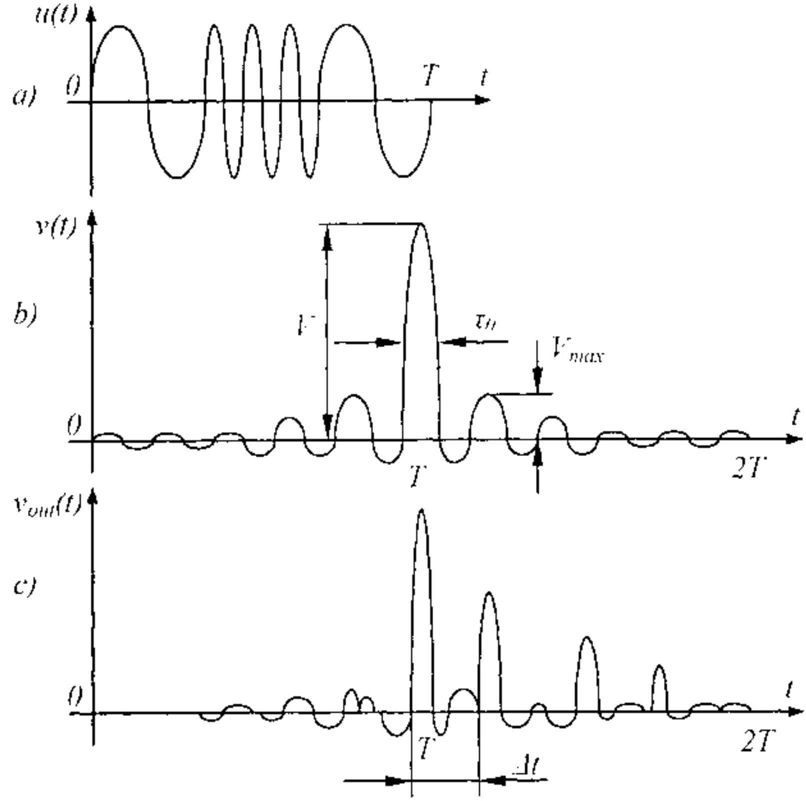

Fig.1. Processing of radar signals

The examined objects reflect the signals, producing so named echo-signals. They are the input to the radar receiver. Usually, in order to maximize the "signal/noise" ratio, the receiver is constructed as a filter, matched to the sent signals. In this case, the receiver's output is the autocorrelation function (*ACF*) of the sent signals. This is clarified in Fig. 1, where a radar signal is shown (Fig. 1a). The duration of the signal is *T*, but it is separated in *n* sub-signals (or "elementary signals") with duration τ_0 (i.e. $n = T / \tau_0$) and different carrier frequency. This technique is named "discrete frequency shift-keying" (*DFSK*).

It allows obtaining a different echo-signal from every "reflected point" of the object. Usually, the receiver's output, produced by a single point echo, is characterized by a main peak V and a sequence of side-lobes with maximal amplitude V_{max}, as shown in Fig. 1b. At the end, the radar receiver output signals are sampled and processed to extract the object image [3].

In general, the above-described technique of using complex radar signals provides for both large performance range (provided by the aggregated power of elementary signals) and high distance resolution (defined by τ_0) of the spacecraft-based synthetic aperture radars (*SARs*). Unfortunately, real objects comprise more than one reflected points. As a result, the echo-signals of all reflected points interfere, as shown in Fig.1c. In this situation, it is hard to obtain a detailed object image, because the side-lobes of the more powerful signals mask the main peaks of the weak signals.

With regard to this, our paper aims to suggest a method for applying the so named generalized complementary signals, whose *ACF* does not have any side-lobes. It uses the polarization features of electromagnetic waves.

2. A Method of Phase-Manipulated Complementary Signals Applied in Spacecraft-Based Radars

It is known [4] that discrete phase- and frequency-modulated signals may be presented as the real part of the complex-valued function:

$$V(t) = \sum_{j=1}^{n} \{U_j . \exp(i\theta_j) . \exp[2\pi i(f_0 + f_j)t]\} . u_0(t - j\tau_0) \quad (1)$$

where $i = \sqrt{-1}$; U_j is the amplitude of the j^th^ elementary pulse $j = 1,2,...,n$; f_0 is the carrier frequency; $\{f_1, f_2, ..., f_n\}$ are real time functions, which express the frequency modulation; $\{\theta_j;\ 0 \le \theta_j < 2\pi;\ j = 1, 2, ...n\}$ is the set of numbers, describing the phase modulation and:

(2) $$u_0(t) = \begin{cases} 1, & if \ \ 0 \le t \le \tau_0 \\ 0, & if \ \ t < 0, \ or \ t > \tau_0 \end{cases}.$$

To maximize transmitter efficiency and to simplify the practical accomplishment of the process of signal receiving, the so-named uniform signals with $\tau_0 = const;\ U_j = const;\ j = 1,2,...,n;;\theta_j \in \{(2\pi l)/m;\ l = 0,1,...,m-1\}$ are most widely applied.

In this case and if only discrete phase shift keying (*DPSK*) is applied, the signal is named "discrete phase-manipulated (*PM*) signal". It can be described precisely by the sequence $\{\zeta(j)\}_{j=1}^{n}$ of normalized complex amplitudes of elementary signals [4]:

(3) $$\zeta(j) = \exp(i.\theta_j);\quad \zeta(j) \in \{\exp(2\pi il/m); l = 0,1,...,m-1\}.$$

As mentioned above, the signals whose *ACF* has close-to-zero level of the side-lobes, are the most attractive for implementation in spacecraft based *SAR*s. With this regard, in the rest part of the paper our attention shall be focused on the so-named generalized complementary signals, whose *ACF* is free of any side-lobes.

It is known that a single radar signal does not have non-periodical *ACF* with zero level of the side-lobes. Moreover, the classes of single uniform discrete radar signals with small level of their side-lobes seem to be very rare. For this reason, Golay introduced [5] the so-named complementary series (or signals (*CS*s)). They are a pair of two uniform binary phase-manipulated signals, whose aggregated non-periodical *ACF* is similar to a delta pulse.

It is necessary to emphasize that Golay's definition of *CS*s is not useful in some important cases. This situation has motivated some theoreticians to extend the classical definition as follows [6, 7, 8].

Definition 1: *The set of p sequences (PM signals), whose elements are complex numbers, belonging to the multiplicative group of the m-th* ($m > 2$) *roots of unity:*

$$\{A_1=\{\xi_1(j)\}_{j=1}^{n_1}; A_2=\{\xi_2(j)\}_{j=1}^{n_2};\dots;A_p=\{\xi_p(j)\}_{j=1}^{n_p}\};$$
$$\xi_k(j)\in\{\exp(2\pi il/m_k); l=0,1,\dots,m_k-1\}; k=1,2,\dots,p. \tag{4}$$

are ***a set of generalized complementary signals*** *(GCCs) if and only if their aggregated ACF has ideal shape, similar to a delta pulse:*

$$R_c(r)=\sum_{k=1}^{p}R_{A_k}(r)=\begin{cases} n=n_1+n_2+\dots+n_p; & if\ \ r=0;\\ 0; & if\ \ \ r=1,2,\dots,\max\{n_k\}.\end{cases} \tag{5}$$

In (5) the non-periodical *ACF* $R_{A_k}(r)$ are defined by the well known formula [4]:

$$R_\zeta(r)=\begin{cases}\sum_{j=1}^{n-|r|}\zeta(j)\zeta^*(j+|r|), & -(n-1)\le r\le 0\\ \sum_{j=1}^{n-r}\zeta^*(j)\zeta(j+k), & 0\le r\le n-1.\end{cases} \tag{6}$$

Consequently, Golay's codes are a particular case of the *GCC*s, when $p=2,\ m=2$. The *CC*s and *GCC*s are unique among all *PM* signals for the following features:

- their aggregated *ACF* has ideal shape, similar to a delta pulse;
- if a pair of *GCC*s, consisting *of n* elements, is known, then it is easy to create an infinite set of pairs with unlimited code-length.

It should be emphasize that most types of uniform *PM* signals with close to ideal *ACF* have limited code-length. For instance, Barker codes exist only for $n\le 13$, if n is an odd integer.

With regard to the *GCCs*' positive features, they are studied very intensively and a quick reference revealed more than 200 conference reports and magazine articles related to this theme during the past ten years.

The natural question, which arises from Definition 1, is "How can the *GCC*s be implemented in a real communication system?". The most obvious answer is to use p different frequency carriers $f_k, k = 1, 2, ..., p$, phase manipulated according to the sequences $A_k, k = 1, 2, ..., p$. Unfortunately, this is not the best approach when the communication system is a spacecraft-based *SAR*. This conclusion will be clarified by the following example. Let us assume that a spacecraft-based *SAR* exploits *GCC*s with $p = 2$ and the transmitter radiates simultaneously two uniform *PM* signals with carriers f_1, f_2, manipulated according to the sequences A_1, A_2. As a result of the so-named Doppler effect, the carriers f_{ek} of the echo-signals will be:

$$f_{ek} = f_k \frac{1 - V_R / c}{1 + V_R / c} \approx f_k (1 - 2V_R / c), \ k = 1, 2, \tag{7}$$

where V_R is the radial velocity of the spacecraft relatively to the object and c is the velocity of the electromagnetic waves propagation. If the explored object is on the earth's surface, then V_R must be at least 27 360 km/h. Then the difference $\Delta f = |f_{e1} - f_{e2}|$ will be too significant and it may lead to irreparable phase distortions between the components of the *GCC*s.

For the above reason, in the rest part of our report we shall prove a more appropriate approach to *GCC*s implementation in spacecraft-based *SAR*s. Namely, we propose the two uniform *PM* signals, composing a pair of *GCC*s, to be transmitted simultaneously on one frequency carrier f_0 but by means of different types of polarization. Let the horizontal and the vertical polarized *PM* signals be described by the sequences $A_1 = \{\mu(j)\}_{j=1}^{n}$ and $A_2 = \{\eta(j)\}_{j=1}^{n}$, respectively. Then the signals, reflected by an object

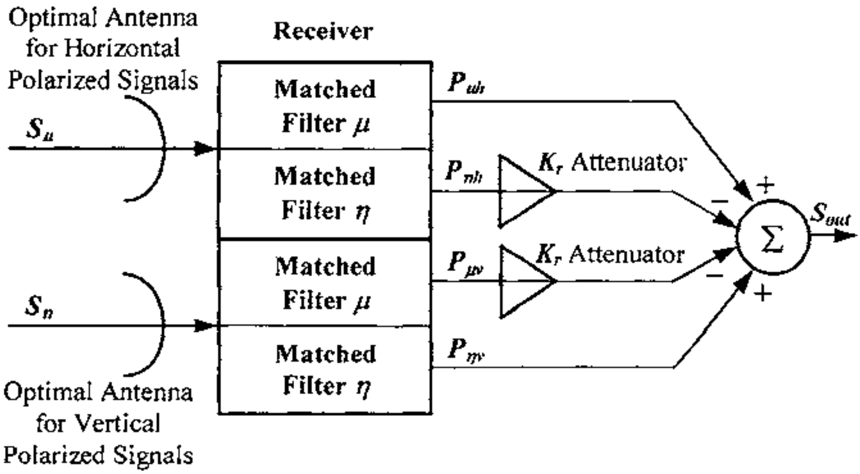

Fig. 2. Method of GCCs applied in spacecraft-based SARs

point, will be S_μ, S_η respectively. The reflected signals are connected with the transmitted signals by the following matrix equation:

$$(8)\qquad \begin{Vmatrix} S_\mu \\ S_\eta \end{Vmatrix} = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix} \begin{Vmatrix} A_1 \\ A_2 \end{Vmatrix}; \ \|D\| = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix},$$

where the complex valued matrix $\|D\|$ is the so-named "polarization matrix of the target scattering". Its entries depend on the physical features of the object, its orientation and position relatively to the radar and the carrier frequency of transmitted signals. When these parameters are constant, then the entries of the matrix $\|D\|$ are also constants and more over, $D_{12} = D_{21}$. Accounting that the size of a single reflected point is small, it may be concluded that $D_{11} \approx D_{22}$. Consequently, the matrix can be presented in the form:

(9) $$\|D\| = \begin{Vmatrix} k_1 & k_1.k_2 \\ k_1.k_2 & k_1 \end{Vmatrix},$$

where $D_{11} = D_{22} = k_1$; $(D_{12}/k_1) = (D_{21}/k_1) = k_2$.

As stated above, the main obstacle for *GCC*s practical implementation by means of polarized electromagnetic waves is the fact that each echo-wave comprises both horizontal and vertical polarized components. For this reason, we propose the method of signal processing, shown in Fig. 2. We shall explain it using the following notation. Let $\{\zeta(j)\}_{j=1}^{n}$ be the sequence of normalized complex amplitudes of elementary signals, composing an arbitrary complex signal. As mentioned, the result of processing this signal by its matched filter will be the *ACF* of the signal. It may be presented by the following polynomial:

(10) $$P(x) = F(x).F^*(x^{-1}).$$

Here:

(11) $$F_\zeta(x) = \zeta(n).x^{n-1} + \zeta(n-1).x^{n-2} + ... + \zeta(2).x + \zeta(1),$$

is the so-named "Hall polynomial", corresponding to the sequence

$\{\zeta(j)\}_{j=1}^{n}$, $F_\zeta^*(x^{-1})$ is the polynomial:

(12) $$F_\zeta^*(x^{-1}) = \zeta^*(n).x^{-(n-1)} + \zeta^*(n-1).x^{-(n-2)} + ... + \zeta^*(1),$$

the coefficients of the polynomial $P(x)$ are:

$$p_k = R_\zeta(k);\ k = -(n-1), -(n-2), ... -1, 0, 1, ..., n-2, n-1,$$

and the values $R_\zeta(k)$ of the *ACF* are computed according to (6).

Accounting for the above notation, (8) and (9), the outputs of the matched filters shown in Fig. 2 may be expressed by the following polynomials:

$$P_{\mu h}(x) = k_1[F_\mu(x) + k_2.F_\eta(x)].F_\mu^*(x^{-1});$$

$$P_{\eta h}(x) = k_1[F_\mu(x) + k_2.F_\eta(x)].[k_r F_\eta^*(x^{-1})];$$

$$P_{\eta v}(x) = k_1[k_2.F_\mu(x) + F_\eta(x)].F_\eta^*(x^{-1}); \tag{13}$$

$$P_{\mu v}(x) = k_1[k_2.F_\mu(x) + F_\eta(x)].[k_r F_\mu^*(x^{-1})].$$

In (13), k_r is a special coefficient, brought into the scheme by means of two directed attenuators. Now it is easy to see, that if the attenuators in Fig.2 are regulated to obtain $k_r = k_2$, then:

$$\begin{aligned} S_{out}(x) &= P_{\mu h}(x) - P_{\eta h}(x) + P_{\eta v}(x) - P_{\mu v}(x) = \\ &= k_1\{[F_\mu(x)F_\mu^*(x^{-1}) + F_\eta(x)F_\mu^*(x^{-1})] - \\ &- k_2^2.[F_\mu(x)F_\mu^*(x^{-1}) + F_\eta(x)F_\mu^*(x^{-1})]\} = k_1.2n.(1-k_2^2). \end{aligned} \tag{14}$$

Formula (14) shows that the method of *GCC*s use, shown in Fig. 2, preserves the cancellation of the *ACF* side-lobes, despite the harmful presence of cross-reflected signals. The signal power losses depend on the relative coefficient of the cross-polarized reflection k_2.

4. Conclusions

The method of *GCC*s applied in spacecraft-based radars, presented above, preserves the positive features of the *GCC*s, especially the cancellation of the *ACF* side-lobes, despite the harmful presence of cross-reflected signals. This result is achieved by small losses of signal power, because mostly $k_2 \le 20\%$ and hence $(1-k_2^2) \ge 96\%$.

References

1. Г е ц о в П., Ж. Ж е к о в, Г. М а р д и р о с я н, И. Х р и с т о в, Ефективност на визирни оптични системи при наблюдение на отдалечени обекти при различна яркост на фона, Сб. научни трудове от НС на ВВУАПВО, ч. 2, Шумен, 1997г., с. 243-249.

2. Ж е к о в Ж., Г. М а р д и р о с я н, И. Х р и с т о в, Д. И в а н о в а. Абсорбционый ультрафиолетовый озонометр, Бълг. геофиз. сп. 1998, № 3 – 4, с. 50 – 54
3. Л а з а р о в, А.Д. Радиовълни, антеннофидерни и микровълнови устройства, БСУ, 2003.
4. В а р а к и н Л. Е. Системы связи с шумоподобными сигналами - М.: Радио и связь, 1985. – 384 с.
5. G o l a y M. Y. E. Complementary series, IRE Trans. on Information Theory, 1961, vol. IT – 7, №2, pp. 82 – 87.
6. T s e n g C. C., S. L. L i u. Complementary sets of sequences, IEEE Trans. on Information Theory, 1972, vol. IT- 18, №5, p. 644 – 652.
7. И г н а т о в В. В., С. А. Д о б р о в о л ь с к и й, А. Ю. Г у ж в а. Матричные системы сигналов для использования в системах CDMA, Электросвязь, 2003, № 9, с. 41-42.
8. F r a n k L.R. Polyphase Complementary Codes, IEEE Trans. on Inform. Theory, 1980, vol. IT-26, № 6, pp. 641 – 647.

МЕТОД ЗА ПРИЛАГАНЕ НА ФАЗОВО МАНИПУЛИРАНИ КОМПЛЕМЕНТАРНИ СИГНАЛИ В КОСМИЧЕСКИТЕ РАДИОЛОКАЦИОННИ СИСТЕМИ

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Резюме

Получаването на радарни изображения на повърхността на планетите, спътниците и кометите е важен момент при тяхното изучаване. В този процес най-важното свойство на автокорелационната функция (АКФ) на радиолокационните сигнали е нивото на страничните листи на АКФ, защото то определя динамичния диапазон на изображението и възможността за откриване на малоразмерни обекти. По тази причина в статията се обосновава метод за изполване на така наречените обобщени комплементарни сигнали (ОКС), чиято сумарна АКФ няма странични листи. Методът се характеризира с това, че запазва ценните свойства на ОКС въпреки ефекта на кръстосано поляризционно отражение на радарните сигнали. Този положитен резултат се постига технически просто и с минимални загуби на енергия на ехо-сигналите.